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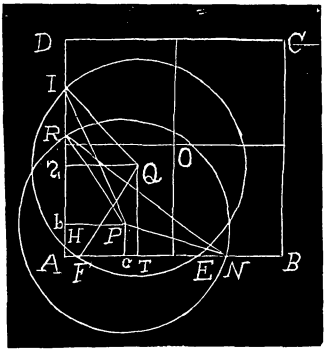
without this quadrant. Coordinates of P, $(Pa, Pb)=(x, y)$; coordinates of Q, $(QT, QS)=(u, v)$.

Then $G=\pi a^2$ − area of segment on IH − area of segment on FE is the area of Q required.



$K=\frac{1}{2}AR\times AN+$ area of segment on RN is the area of P required.

Let $\angle TQF=\theta$, $\angle IQS=\phi$, $\angle NPa=\psi$, $\angle RPb=\rho$.

$\therefore u=a\cos\theta$, $v=a\cos\phi$, $x=a\cos\psi$, $y=a\cos\rho$, $AN=a\cos\rho+a\sin\psi$, $AR=a\cos\psi+a\sin\rho$, $\angle RPN=3\pi/2-\psi-\rho$.

$\therefore G=(\pi-\theta-\phi+\sin\theta\cos\theta+\sin\phi\cos\phi)a^2$;

$K=\frac{1}{2}(3\pi/2-\psi-\rho+\cos\rho\sin\rho+\cos\psi\sin\psi+2\cos\psi\cos\rho)a^2$.

The limits of θ are $\frac{1}{2}\pi-\phi$ and 0; of ϕ, 0 and $\frac{1}{2}\pi$; of ψ, $\frac{1}{2}\pi-\rho$ and $\frac{1}{2}\pi$; of ρ, 0 and $\frac{1}{2}\pi$. Then Δ, the required average area, is

$$\Delta=\frac{\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi-\phi}Ga^2\sin\phi\sin\theta\,d\phi\,d\theta+\int_0^{\frac{1}{2}\pi}\int_{\frac{1}{2}\pi-\rho}^{\frac{1}{2}\pi}Ka^2\sin\rho\sin\psi\,d\rho\,d\psi}{\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi-\phi}a^2\sin\phi\sin\theta\,d\phi\,d\theta+\int_0^{\frac{1}{2}\pi}\int_{\frac{1}{2}\pi-\rho}^{\frac{1}{2}\pi}a^2\sin\rho\sin\psi\,d\rho\,d\psi}$$

$$=\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi-\phi}G\sin\phi\sin\theta\,d\phi\,d\theta+\int_0^{\frac{1}{2}\pi}\int_{\frac{1}{2}\pi-\rho}^{\frac{1}{2}\pi}K\sin\rho\sin\psi\,d\rho\,d\psi$$

$$=\tfrac{1}{6}a^2\int_0^{\frac{1}{2}\pi}[6(\pi-\phi+\sin\phi\cos\phi)\sin\phi-3\sin^3\phi\cos\phi-2\cos^3\phi\sin\phi-2\sin\phi]\,d\phi$$

$=(\pi-\frac{29}{24})a^2$, ϕ and ρ having the same limits, $\phi=\rho$.

MISCELLANEOUS.

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155 (Incorrectly numbered 151). Proposed by W. J. GREENSTREET, M. A., Stroud, England.

Sum the series $\sum_{r=1}^{r=m}\operatorname{cosec}\left(\frac{2r-1}{4m}\pi+\theta\right)\operatorname{cosec}\theta\left(\frac{2r-1}{4m}\pi-\theta\right)$.

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

$$S=\sum_{r=1}^{r=m}\operatorname{cosec}\left(\frac{2r-1}{4m}\pi+\theta\right)\operatorname{cosec}\left(\frac{2r-1}{4m}\pi-\theta\right)$$

$$= \sum_{r=1}^{r=m} \frac{\sin\left[\frac{2r-1}{4m}\pi + \theta - \left(\frac{2r-1}{4m}\pi - \theta\right)\right]}{\sin 2\theta \sin\left(\frac{2r-1}{4m}\pi + \theta\right) \sin\left(\frac{2r-1}{4m}\pi - \theta\right)}$$

$$= \operatorname{cosec} 2\theta \sum_{r=1}^{r=m} \left[\cot\left(\frac{2r-1}{4m}\pi - \theta\right) - \cot\left(\frac{2r-1}{4m}\pi + \theta\right)\right]$$

$$= \operatorname{cosec} 2\theta \sum_{r=1}^{r=m} \left[\cot\left(\frac{2r-1}{2m}\pi - 2\theta\right) - \cot\left(\frac{2r-1}{2m}\pi + 2\theta\right)\right.$$

$$\left. + \operatorname{cosec}\left(\frac{2r-1}{2m}\pi - 2\theta\right) - \operatorname{cosec}\left(\frac{2r-1}{2m}\pi + 2\theta\right)\right].$$

But $\operatorname{cosec}\left(\frac{2r-1}{2m}\pi \mp 2\theta\right) = \operatorname{cosec}\left(\frac{2m-2r+1}{2m}\pi \pm 2\theta\right)$.

$$\therefore \sum_{r=1}^{r=m} \left[\operatorname{cosec}\left(\frac{2r-1}{2m}\pi - 2\theta\right) - \operatorname{cosec}\left(\frac{2r-1}{2m}\pi + 2\theta\right)\right] = 0.$$

$$\therefore S = \operatorname{cosec} 2\theta \sum_{r=1}^{r=m} \left[\cot\left(\frac{2r-1}{2m}\pi - 2\theta\right) - \cot\left(\frac{2r-1}{2m}\pi + 2\theta\right)\right].$$

From trigonometry, $\prod_{r=0}^{r=m-1} \sin\left(\frac{r\pi}{m} + \phi\right) = \frac{\sin m\phi}{2^{m-1}}$.

Differentiating both sides, and reducing, we get,

$$\sum_{r=0}^{r=m} \cot\left(\frac{r\pi}{m} + \phi\right) = m \cot m\phi.$$

Let $\phi = \frac{\pi}{2m} - 2\theta$, and also $\frac{\pi}{2m} + 2\theta$. Then we get,

$$S = \operatorname{cosec} 2\theta \sum_{r=1}^{r=m} \left[\cot\left(\frac{2r-1}{2m}\pi - 2\theta\right) - \cot\left(\frac{2r-1}{2m}\pi + 2\theta\right)\right]$$

$$= m \operatorname{cosec} 2\theta \left[\cot m\left(\frac{\pi}{2m} - 2\theta\right) - \cot m\left(\frac{\pi}{2m} + 2\theta\right)\right] = 2m \operatorname{cosec} 2\theta \tan 2m\theta.$$

Also, $\sum_{r=1}^{r=m} \sec\left(\frac{2r-1}{4m}\pi + \theta\right) \sec\left(\frac{2r-1}{4m}\pi - \theta\right) = 2m \operatorname{cosec} 2\theta \tan 2m\theta = S$ above.

This follows because

$$\sec\left(\frac{2r-1}{4m}\pi \pm \theta\right) = \operatorname{cosec}\left(\frac{2m-2r+1}{4m}\pi \mp \theta\right).$$

PROBLEMS FOR SOLUTION.

GEOMETRY.

333. Proposed by J. B. MORRELL, Boulder, Colorado.

Exhibit the fallacious argument to prove that a right-angle is equal to an angle which is less than a right-angle.

334. Proposed by J. O. MAHONEY, B. E., M. Sc., Central High School, Dallas, Texas.

Through any point P in the plane of the triangle ABC, draw a line that shall divide the perimeter of the triangle into two equal parts.

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

152. Proposed by H. S. VANDIVER, Bala, Pa.

Prove geometrically:

$$\sum_{n=1}^{\frac{1}{2}(p-1)}\left[\frac{n^2}{p}\right] = \frac{p-3}{4}\cdot\frac{p-1}{2} - \sum_{n=1}^{\frac{1}{2}(p-4)}\left[\sqrt{np}\right],$$

where $p \equiv 3 \pmod 4$ and $\left[\frac{k}{p}\right]$ represents the greatest integer in k/p.

AVERAGE AND PROBABILITY.

196. Proposed by R. D. CARMICHAEL, Anniston, Ala.

A circle is inscribed in a square. Find the chance that the distance between two points within the square and without the circle shall not exceed a side of the square.

197. Proposed by HENRY HEATON, Belfield, N. D.

Solve No. 188 on the supposition that all lines having the same direction are equally distributed in space, and lines passing through the same point are distributed as the radii of a sphere drawn to points equally distributed.

MISCELLANEOUS.

177. Proposed by R. D. CARMICHAEL, Anniston, Ala.

Sum the infinite series:

(a) $\sin x + nx\cos x - \frac{n^2 x^2}{2!}\sin x - \frac{n^3 x^3}{3!}\cos x + \frac{n^4 x^4}{4!}\sin x + \ldots,$

(b) $\cos x - nx\sin x - \frac{n^2 x^2}{2!}\cos x + \frac{n^3 x^3}{3!}\sin x + \frac{n^4 x^4}{4!}\cos x \ldots$